beginning to decay, on the 15th. These three month dates refer, of course, to the conjunction, first appearance, and plenitude of the moon. The 16th of the lunar month was oftener than the 15th regarded as the day of full moon; and two monthly festivals, celebrated on the 1st and 16th, have been rightly referred by Lepsius to the lunar year.

Dates of the lunar year are, according to Dr. Hincks, occasionally met with on the monuments. Such dates are, he thinks, those of the 1st and 16th of Athyr, of the 11th of Amenhotap III., mentioned on a scarabæus, so as to imply that the Nile was then rising, and near its height. The date of the Exodus in the month Abib, presumably Epiphi, is also referred to a lunar year. The month Abib is identified with what was afterwards the first month of the Israelites, and was, therefore, like this, a lunar month; and we know that it was the month which began at the new moon following the vernal equinox. Epiphi was the eleventh Egyptian month; the lunar Epiphi would, therefore, begin more than 295 days after the solstice, while the vernal equinox was about 271 days after it. From this it follows that the first Hebrew month would in general coincide with Payni, the tenth Egyptian month; but that it would occasionally coincide with Epiphi—namely, when the new moon followed the summer solstice very closely. This would furnish a means of determining the year of the Exodus accurately, if it were known approximately; for example, 1491 B. C. could not be the year of the Exodus, but 1494 B. C. might. But, what is of more consequence, the remark respecting the month Abib is a very strong argument in favour of the genuineness of the Biblical account of the Exodus, which has been recently called in question. No forger of a later age, and who had not lived in Egypt, could have thought of making such a statement.

The President read a paper, by Professor Sylvester, "On the Demonstration of Newton's Theorem respecting the Imaginary Roots of Equations."

The President read the following paper, with a Note by the late Sir W. R. Hamilton, LL. D.:—

On a Theorem relating to the Binomial Coefficients.

Towards the end of March, I communicated the following theorem to Sir William Rowan Hamilton:—

$$\text{Putting } s_0 = n_0 + n_3 + n_6 + \&c.,$$
$$s_1 = n_1 + n_4 + n_7 + \&c.,$$
$$s_2 = n_2 + n_5 + n_8 + \&c.,$$

where n_0, n_1, &c., are the coefficients of the development

$$(1 + x)^n = n_0x^0 + n_1x^1 + n_2x^2 + \&c.,$$

and n is a positive whole number;

we shall find that, of the three quantities, s_0, s_1, s_2, two are always equal, and the third differs from them by unity.

I mentioned at the same time that I had arrived at theorems, analogous, but less elegantly expressed, by summing the series formed by taking every *fourth* or *fifth* coefficient, and so on, in the binomial development; and I asked Sir William R. Hamilton whether he remembered to have seen these theorems stated anywhere. I thought it likely that the well-known elementary theorem respecting the equality of the sums of the alternate coefficients in the binomial development would have suggested research in this direction. In a note, written on the day on which he received mine, Sir William stated that my theorem was new to him, and that he had proved it by the help of imaginaries and determinants. The following day he wrote again to me, furnishing me with the following more precise statement of my theorem:—

"Let ν and N be the following (whole) functions of n,

$$\nu = (-1)^n, \; N = \tfrac{1}{3}(2^n - \nu);$$

then N, N and $N+\nu$ are *always* the value of the *three sums*, if suitably *arranged;* and the *singular sum* is s_0, or s_1, or s_2, according as n, or $n+1$, or $n+2$ is a multiple of 3."

I communicated the following demonstration of my theorem to Sir William, in a letter of the 29th March:—

Using the notation employed above, we know that

$$(n+1)_r = n_r + n_{r-1},$$
$$(n+1)_{r-1} = n_{r-1} + n_{r-2},$$
$$\text{and } (n+1)_r - (n+1)_{r-1} = n_r - n_{r-2}.$$

Now, putting $s_r = \ldots\ldots + n_{r-m} + n_r + n_{r+m} + \ldots\ldots,$

$$s'_r = \ldots\ldots + (n+1)_{r-m} + (n+1)_r + (n+1)_{r+m} + \ldots\ldots,$$

(m being any positive integer),

we have, from equation (1),

$$s'_r - s'_{r-1} = s_r - s_{r-2};$$

and, in the particular case under consideration, viz. $m = 3$,

$$s'_2 - s'_1 = s_2 - s_0,$$
$$s'_1 - s'_0 = s_1 - s_2,$$
$$s'_0 - s'_2 = s_0 - s_1.$$

Thus it appears that the *differences* of the quantities s'_0, s'_1, s'_2, are equal in magnitude, but of opposite signs to those of s_1, s_2, s_0; and if we form these differences for successive values of n, they will arrange themselves in a cycle of six. Thus, if

$$s_2 - s_1 = \Delta_0,\ s_1 - s_0 = \Delta_2,\ s_0 - s_2 = \Delta_1,$$

we might form the following Table.

n	Δ_0	Δ_2	Δ_1
1	-1	0	1
2	-1	1	0
3	0	1	-1
4	1	0	-1
5	1	-1	0
6	0	-1	1
7	-1	0	1
8	-1	1	0
. .	. .	. .	. .

Combining this result with the well-known theorem—

$$s_0 + s_1 + s_2 = n_0 + n_1 + n_2 + \ldots\ldots = 2^n,$$

we arrive at formulæ for s_0, s_1, s_2.

A couple of days later, I communicated to Sir William Hamilton my statement and proof of the corresponding theorem respecting the four sums obtained by adding every fourth binomial coefficient.

The theorem is as follows :—

" Writing $\nu = (-1)^i$ where i is any positive integer,

If n is of the form $4i$,

$$s_0 = 2^{n-2} + \nu\, 2^{\frac{n-2}{2}},$$
$$s_1 = 2^{n-2},$$
$$s_2 = 2^{n-2} - \nu\, 2^{\frac{n-2}{2}},$$
$$s_3 = 2^{n-2};$$

If n is of the form $4i + 1$,

$$s_0 = 2^{n-2} + \nu\, 2^{\frac{n-3}{2}},$$
$$s_1 = 2^{n-2} + \nu\, 2^{\frac{n-3}{2}},$$
$$s_2 = 2^{n-2} - \nu\, 2^{\frac{n-3}{2}},$$
$$s_3 = 2^{n-2} - \nu\, 2^{\frac{n-3}{2}};$$

If n is of the form $4i + 2$,

$$s_0 = 2^{n-2},$$

$$s_1 = 2^{n-2} + \nu\, 2^{\frac{n-2}{2}},$$

$$s_2 = 2^{n-2},$$

$$s_3 = 2^{n-2} - \nu\, 2^{\frac{n-2}{2}};$$

If n is of the form $4i + 3$,

$$s_0 = 2^{n-2} - \nu\, 2^{\frac{n-3}{2}},$$

$$s_1 = 2^{n-2} + \nu\, 2^{\frac{n-3}{2}},$$

$$s_2 = 2^{n-2} + \nu\, 2^{\frac{n-3}{2}},$$

$$s_3 = 2^{n-2} - \nu\, 2^{\frac{n-3}{2}}.\text{''}$$

The proof of this rests upon the equations

$$s'_3 - s'_2 = s_3 - s_1,$$
$$s'_2 - s'_1 = s_2 - s_0,$$
$$s'_1 - s'_0 = s_1 - s_3,$$
$$s'_0 - s'_3 = s_0 - s_2,$$

combined with $s_0 + s_1 + s_2 + s_3 + 2^n$.

Though the theorems which I have now stated or indicated are not devoid of interest, I should hardly have brought them under the notice of the Academy if they had not led Sir William R. Hamilton to discuss the more general question treated of in the Note appended to this paper. It is at his suggestion that I have communicated the substance of the letter which I addressed to him on this subject.

I may be allowed to add, that the first theorem stated in this paper was suggested by the investigation of a very simple geometrical problem, and that I have found that it admits of being very curiously illustrated by means of my theory of algebraic triplets.

EXTRACT *from a recent Manuscript Investigation, suggested by a Theorem of* DEAN GRAVES, *which was contained in a Letter received by me a week ago.*

1. Let n_r, for any whole value not less than zero of n, and for *any* whole value of r, be defined to be the (always whole) coefficient of the power